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THE DISTURBANCE CAUSED IN TEMPERATURE DUE TO PRESENCE OF THREE INTERIOR GRIFFITH-CRACKS IN AN ISOTROPIC RECTANGLE

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ABSTRACT

The Closed form expression of disturbance in temperature distribution in a rectangular isotropic plate in the presence of three interior Griffith Cracks is being obtained by using the principle of cross-linear-superposition along with Fredholm integral equation. It is found that temperature distribution at crack tips is smooth. Flux possesses Cauchy type singularity at crack tips.

KEY WORDS: Flux intensity factor, Fredholm integral, Cross-linear Superposition.

INTRODUCTION

we considered the problems of single and double Griffith-cracks and got dual and triple series equations which were solved by the method of Parihar [1].It is very obvious that what will happen if there are three Griffith-cracks in the isotropic rectangle. As we know this problem will reduce to quadruple series equations. We consider a cross-section of three dimensional body having three Griffith-cracks along x-axis and y-axis being through the midpoint of middle crack and perpendicular to x-axis. Thus we consider a rectangle of length $2a$ and width 2δ . In previous chapters width was (δ_1 and δ_2). x axis is along length of rectangle and y-axis along width of rectangle.[2] Cracks occupy the region $y= 0$, $0\leq x < b$, $d \leq x < e < a$,. The physical problem will be reduced to the following boundary value problem for steady case.

$$\frac{\partial}{\partial y}T(x,\delta) = \frac{\partial}{\partial y}T(x,\pm\delta) = Q_1(x) \quad 0 \le x \le a \tag{1.1}$$

$$\frac{\partial}{\partial x}T(\pm a, y) = Q_2(y) \quad ,0 \le |y| \le \delta \tag{1.2}$$

$$\frac{\partial}{\partial y}T(x,0) = Q_3(x) \ , 0 \le |x| \le b \ , d < |x| < e < a \tag{1.3}$$

$$T(x,0) = 0 \quad ,b \le |x| \le d \quad ,e \le |x| \le a \tag{1.4}$$

It is being assumed that specific heat and linear expansion do not very with heat change. The temperature distribution is governed by following Laplace equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)T(x, y) = 0 \tag{1.5}$$

The plan of paper is as follows: We will formulate and reduce the problem to quadruple series equations. The solution of quadruple series equation will be obtained. Further we will solve the physical quantities in terms of solution of fredholm integral equation. The solution of Fredholm integral equation will be given in section.

FORMULATION

For the solution of (1.5) we use the principle of cross-linear superposition along with finite Fourier transform method. We consider the solution as

$$T(x,y)=\frac{A_0+C_0}{2}+\sum_{n=1}^{\infty}\left[A_n\cosh\alpha_n y+B_n\sinh\alpha_n y\right]\cos\alpha_n x+\sum_{m=1}^{\infty}\left(C_m\cosh\beta_m x\right)\cos\beta_m y \qquad 2.1$$

Where A_n , $B_{n,}C_n$ are three constants to be determined and $\alpha_n=\frac{n\pi}{a}$, $\beta_m=\frac{m\pi}{\delta}$ ……… 2.2

DETERMINATION OF CONSTANTS

The boundary conditions in (1.1) and (1.2), after using symmetry of geometry and using (2.1), will give

$$\sum_{n=1}^{\infty}\alpha_n\left[A_n\sinh\alpha_n\delta+B_n\cosh\alpha_n\delta\right]\cos\alpha_n x=Q_1(x) \quad ,0\le x\le a \qquad 2.3$$

$$\sum_{m=1}^{\infty}\beta_m C_n\sinh\beta_m a\cos\beta_m y=Q_2(y) \qquad 0\le y\le\delta \qquad 2.4$$

The boundary conditions (1.4) along with (2.1) , gives

$$\frac{A_0+C_0}{2}+\sum_{n=1}^{\infty}A_n\cos d_n x+\sum_{m=1}^{\infty}C_m\cosh\beta_m x=0 \quad ,b\le x<d\,,e\le x\le a \qquad 2.5$$

The form 2.3 -2.4 we get inversion of finite Fourier Transforms

$$\alpha_n\left[A\sinh(\alpha_n\delta)+B_n\cosh(\alpha_n\delta)\right]=F_1(\alpha_n) \qquad 2.6$$

$$\beta_m C_m\sinh\beta_m\alpha=F_2(\beta_m) \qquad 2.7$$

$$F_1(\alpha_n)=\int_0^a Q_1(x)\cos(\alpha_n x)dx \quad and \;\; F_2(\beta_m)=\int_0^b Q_2(y)\cos(\beta_m y)dy \qquad 2.8$$

Using 2.7 in 2.5 , we get

$$\frac{A_0+C_0}{2}+\sum_{n=1}^{\infty}A_n\cos d_n x=-P_0(x) \quad ,b\le x\le d\,,e\le x\le a \qquad 2.9$$

$$P_0(x)=\sum_{n=1}^{\infty}\frac{F_2(\beta_m)}{\beta_m}\cosh(\beta_m a)\cosh(\beta_m x) \qquad 2.10$$

The boundary condition 1.3 with 2.1 gives

$$\sum_{n=1}^{\infty}\alpha_n\beta_m\cos(\alpha_n x) \;\; =Q_1(x) \quad ,0\le x<b\,,d<x<e \qquad 2.11$$

Using 2.6 in 2.11 which gives

$$\sum_{n=1}^{\infty}\alpha_n A_n\cos(\alpha_n x) \;\; =P_3(x) \quad ,0\le x<b\,,d\le x<e \qquad 2.12$$

$$P_3(x) \;\; =Q_3(x)+P_1(x)+P_2(x)$$

$$P_1(x)=\sum_{n=1}^{\infty}\frac{F_1(\alpha_n)\cos(\alpha_n x)}{\cosh(\alpha_n\delta)} \quad and \;\; P_2(x)=\sum_{n=1}^{\infty}\frac{\alpha_n A_n e^{-a_n\delta}}{\cosh(\alpha_n\delta)}\cos(\alpha_n x) \qquad 2.13$$

Thus the physical problem is reduced to the solution of quadruple series equation (2.10) and (2.12).

SOLUTION OF QUADRUPLE SERIES EQUATION

We shall follow the method of Kushwaha [15]. We assume the trial solution as−

$$A_n = \frac{2}{q}\left[\left(\int_0^b g(t) + \int_d^e h(t)\right)\frac{\sin(\alpha_n t)}{n}dt + a^{-1}\left\{\int_0^d - \int_c^a\right\}P_0'(t)\frac{\sin\alpha_n}{n}dt\right] \quad 3.1$$

$$\frac{A_0 + C_0}{2} = \left[\left(\int_0^b tg(t) + \int_d^e th(t_1)dt\right) + a\left(\left(\int_b^d - \int_c^a\right)tP_0'(t)dt\right) - P_0(a)\right] \quad 3.2$$

The substitution of (3.1) and (3.2) into 2.5 will satisfy it if

$$\int_d^e h(t)dt = \frac{1}{a}\left[P_0(e) - P_0(d)\right] \quad 3.3$$

Now we substitute from 3.1 into 2.11 , we get

$$g(t) = \frac{2}{a}\frac{\sin(qt/2)}{\psi(t)}\left[\Delta_0(t) + \left\{\int_0^b g(\alpha) - \int_d^e h(\alpha)\right\}K(\alpha,t)d\alpha\right] \quad , 0 \le t \le b \quad 3.4$$

$$\Delta_0(t) = \Delta_{01}(t) + \Delta_{02}(t) + \Delta_{03}(t)$$

$$\left.\begin{aligned}
\Delta_{01}(t) &= \left(\int_0^b - \int_d^e\right)\frac{\left[Q_3(x)\cos\left(\frac{qx}{2}\right)\psi(x)\right]}{G(x,t)} + D_0 \quad , 0 \le t \le b \\
\Delta_{02}(t) &= \left(\int_0^b - \int_d^c\right)\frac{\left[P_1(x)\cos\left(\frac{qx}{2}\right)\psi(x)\right]}{G(x,t)}dx \\
\Delta_{03}(t) &= \left(\int_0^b - \int_c^a\right)\frac{\left[P_0'(\alpha)\sin(q\alpha)\right]}{G(\alpha,t)}dx
\end{aligned}\right] \quad 3.5$$

$$K(\alpha,t) = \left(\int_0^b - \int_d^e\right)\frac{\left[\cos\left(\frac{qx}{2}\right)\psi(x)M(\alpha,x)\right]}{G(x,t)}dx \quad 0 \le t \le b \quad 3.6$$

$$M(y,x) = \sum_{n=1}^{\infty}\frac{e^{-\alpha_n\delta}\cos(\alpha_n x)\sin(\alpha_n y)}{\cosh(\alpha_n\delta)} \quad 3.7$$

$$h(t) = \frac{-2}{a}\frac{\sin\left(\frac{qt}{2}\right)}{\psi(t)}\left[\Delta_0(t) + \left(\int_0^b g(\alpha) - \int_d^e h(\alpha)\right)K(\alpha,t)d\alpha\right] \quad 3.8$$

With

$$\psi(x) = \left[|G(x,b)||G(x,d)||G(x,e)|\right]^{1/2} \quad 3.9$$

$$G(x,y) = \cos(qx) - \cos(qy) \quad , q = \frac{\pi}{2} \quad 3.10$$

And D_0 is an arbitrary constant to be determined by (3.3). For first approximation we take

$$h(t) = \frac{-2}{a} \frac{\sin\left(\frac{qt}{2}\right)}{\psi(t)} \Delta_{01}(t) \quad , d \le t \le e \tag{3.11}$$

$$D_0 = \frac{P_0(d) - P_0(e) - 2t_2}{2t} \tag{3.12}$$

$$\left.\begin{aligned} t_1 &= \frac{2\sec(qd/2)\sec(qe/2)}{q} t_3 \\ t_2 &= -2\int_0^b Q_3(x)\cos(qx/2)\,dx \\ t_3 &= \int_0^{\sin(\frac{qb}{2})} \frac{dt}{\left[\left(\alpha^2+t^2\right)\left(\beta^2+t^2\right)\right]^{\frac{1}{2}}} \end{aligned}\right] \tag{3.13}$$

$$\alpha = \frac{G(b,d)}{1+\cos(qd)} \quad , \quad \beta = \frac{G(b,e)}{1+\cos(qe)}$$

PHYSICAL QUANTITIES

In this section we shall evaluate flux and Temperature for $y = 0$.

T E M P E R A T U R E

Temperature $T(x,0)$, $0 \le x \le b$, $d \le x \le e$ will be obtained through the equation (2.9) with $P_0(x)$ on left hand side. Thus

$$T(x,0) = \frac{\pi}{2}\int_x^b g(t) - P_0(b) + P_0(x) \quad ,0 \le x < b \tag{4.1}$$

And

$$T(x,0) = \frac{\pi}{2}\int_x^b h(t) - P_0(e) + P_0(x) \quad ,d < x < e \tag{4.2}$$

Where $g(t)$ *and* $h(t)$ are the solution of coupled Fredholm integral equation of second kind given by (3.4) and (3.8) , respectively . The temperature at general point (x,y) is given as ,

$$T(x,y) = \frac{A_0 + C_0}{2}\left[\left(\int_0^b g(t) + \int_d^b h(t)dt\right) + \frac{1}{a}\left(\left(\int_b^d - \int_e^a\right) P_0^{'}(t)dt\right)\right]$$
$$N_1(x,y,t) + P_4(x,y) + P_5(x,y) \quad , 0 \le x \le a \,, 0 < y \le \delta \tag{4.3}$$

$$N_1(x,y,t)=\sum_{n=1}^{\infty}\frac{\left[\cosh(\alpha_n y)-\alpha_n\tanh(\alpha_n\delta)\sinh(\alpha_n y)\right]}{\alpha_n}\cos(\alpha_n x)\sin(\alpha_n t) \qquad 4.4$$

$$P_4(x,y)=\sum_{n=1}^{\infty}F_1\frac{(\alpha_n\delta)\sinh(\alpha_n y)\cos(\alpha_n x)}{\cosh(\alpha_n\delta)} \qquad 4.5$$

$$P_5(x,y)=\sum_{m=1}^{\infty}F_2\frac{(\beta_m)\cosh(\beta_m x)\cos(\beta_m y)}{\beta_m\sinh(\beta_m a)} \qquad 4.6$$

FLUX

The flux is obtained for $y=0$, $b\le x\le d$, $e\le x\le a$ from the equation (2.12), keeping $Q_3(x)$ on right hand side and taking $P_1(x)$ and $P_2(x)$ on left hand side as

$$\frac{\partial}{\partial y}T(x,0)=-\left[a\Delta_3(x)+P_1(x)+\Delta_4(x)\right],\ b\le x\le d \qquad 4.7$$

$$\frac{\partial}{\partial y}T(x,0)=\ a\Delta_3(x)-P_1(x)-\Delta_4(x),\ e\le x\le a \qquad 4.8$$

$$\Delta_3(x)=\left[\left(\int_0^b g(t)+\int_d^e h(t)+a^{-1}\left(\int_b^d-\int_e^a\right)P_0'(t)\right)\frac{\sin(qt)}{G(x,t)}dt\right] \qquad 4.9$$

$$\Delta_4(x)=\left[\left(\int_0^b g(t)+\int_d^e h(t)+a^{-1}\left(\int_b^d-\int_e^a\right)P_0'(t)\right)M_1(x,t)dt\right] \qquad 4.10$$

$$M_1(x,t,q_1)=M_{11}(x,t;2q_1)-M_{12}(x,t;4q_1)$$

$$M_{1i}(x,t;2iq_1)=\frac{1}{2}\tanh(2iq_1)+\frac{1}{2}\frac{\tanh(2iq_1)}{\cosh(2iq_1)}\cos(qt)\cos(qx)-\sinh(2iq_1)\left(1+\cos qt\cos 2qx\right)-$$

$$\frac{\operatorname{sech}^2(2iq_1)}{4}\left(\cos(qt)+\cos(qx)+\cos(qx)\cos 2qt+\cos(qt)\cos 3qx\right) \qquad 4.11$$

$$q_1=q\delta=\frac{\pi\delta}{a};\ i=1,2$$

Flux has Cauchy type singularity at crack tips.

FLUX INTENSITY FACTOR

We define flux intensity factor as

$$K_b=\lim_{x\to b^+}\sqrt{x-b}\,\frac{\partial}{\partial y}T(x,0)$$

$$K_d=\lim_{x\to d^-}\sqrt{d-x}\,\frac{\partial}{\partial y}T(x,0)$$

$$K_e=\lim_{x\to e^+}\sqrt{x-e}\,\frac{\partial}{\partial y}T(x,0) \qquad 4.12$$

Making use of (4.12) in (4.7)-(4.10) and evaluating the limits. We get,

$$K_b=a\Delta_3(b),\ K_d=-a\Delta_3(d),\ K_e=a\Delta_3(e) \qquad 4.13$$

While we kept in mind that the terms $P_1(x)$ and $\Delta_3(x)$ do not possess singularity at crack tips.

SOLUTION OF FREDHOLM INTEGRAL EQUATION

We will solve the coupled Fredholm integral equation by approximate expansion method in terms of $\left\{e^{nq\delta}\right\}$,for different n = 1, 2, 3, ... We retained up to $e^{-6q\delta}$ only. Before we go to explain this method first we take the boundary conditions and the functions involved in the solution of $g(t)$ and $h(t)$ interms of Kernel $K(\alpha,t)$. We assume that the faces $x=\pm a$ posses' constant flux and the face $y=\pm\delta$ are insulated. Thus

$$Q_1(x)=0\ ,Q_2(y)=d_1\ and\ Q_3(y)=d_2\quad d_1\ ,d_2\ are\ \text{Constants} \qquad 5.1$$

Now making use of (5.1) and (2.8) we get

$$F_1(\alpha_n)=0\quad ,F_2(\beta_m)=0 \qquad 5.2$$

Then

$$P_0(x)=0\ ,P_1(x)=0\ ,D_0=-t_2/t_1=\frac{-2}{q}d_2\frac{\sin(qb/2)}{t_1} \qquad 5.3$$

Further from (3.5)

$$\Delta_{01}(t)=d_2\left[a_1+\cos(qt)a_2+\cos^2(qt)a_3\right]+D_0 \qquad 5.4$$

Where

$$a_1=\left(\int_0^b-\int_d^e\right)\frac{\cos(qx/2)\sqrt{|G(x,d|G(x,e)}}{\sqrt{|G(x,b)|}}dx \qquad 5.4.a$$

$$a_2=\left(\int_0^b-\int_d^e\right)\frac{\cos(qx/2)\sqrt{|G(x,e|}}{\sqrt{|G(x,b)||G(x,d)|}}dx \qquad 5.4.b$$

$$a_3=\left(\int_0^b-\int_d^e\right)\frac{\cos(qx/2)}{\psi(x)}dx \qquad 5.4.c$$

$$\Delta_{02}(t)=0 \qquad 5.5$$

The approximate expansion for $M(\alpha,x)\ given\ for\ K(\alpha,x)\ as,$

$$K(\alpha,x)=\left(\int_0^b-\int_d^e\right)P(x,t)\left[2e^{-2q_1}\psi_0(\alpha,x)+2e^{-4q_1}\psi_1(\alpha,x)-3e^{-6q_1}\psi_2(\alpha,x)\right]dx \qquad 5.6$$

$$P(x,t)=\frac{\cos(qx/2)\psi(x)}{G(x,t)} \qquad 5.6.a$$

$$\psi_0(\alpha,x)=1+\cos(q\alpha)\cos(qx)+\cos(2q\alpha)\cos(2qx) \qquad 5.6.b$$

$$\psi_1(\alpha,x)=-3-\cos(q\alpha)\cos(qx)+2\cos(2q\alpha)\cos(2qx)+2\cos(q\alpha)+2\cos(qx)$$
$$+2\cos(qx)\cos(2q\alpha)+2\cos(2qx)\cos(qx) \qquad 5.6.c$$

$$\psi_2(\alpha,x)=2\psi_0(\alpha,x) \qquad 5.6.d$$

Now we assume that

$$g(t)=\sum_{n=0}^{\infty}g_n(t)e^{-nq_1}\quad ,\ h(t)=\sum_{m=0}^{\infty}h_m(t)e^{-mq_1} \qquad 5.7$$

And substitute in (3.4) and (3.8) and comparing the coefficient of $\left\{e^{-nq_1}\right\}$ from both sides and we took n=6

$$g_0(t)=\frac{2}{a}\psi_5(t) \quad ,\psi_5(t)=r(t)\Delta_{01}(t) \quad ,r(t)=\frac{\sin(qt/2)}{\psi(t)} \qquad 0<t<b \tag{5.8.a}$$

$$h_0(t)=-\frac{2}{a}\psi_5(t) \quad , \quad d<t<e \tag{5.8.b}$$

$$\left.\begin{array}{l} g_1(t)=g_3(t)=g_5(t)=0 \quad , 0\le t<b \\ h_1(t)=h_2(t)=h_5(t)=0 \quad , d<t<e \end{array}\right]$$

$$\left.\begin{array}{l} g_2(t)=\dfrac{2}{a}\psi_6(t) \quad ,\psi_6(t)=r(t)\psi_{61}(t) \\ \psi_{61}(t)=\left(\displaystyle\int_0^b+\int_d^e\right)r(\alpha)\{\Delta_0(x)\phi_0(\alpha,t)\}d\alpha \qquad ,0\le t<b \\ h_2(t)=-\dfrac{2}{a}\psi_6(t) \qquad , d<t<e \end{array}\right]$$

$$\left.\begin{array}{l} g_4(t)=\dfrac{2}{a}\psi_7(t) \quad ,\psi_7(t)=r(t)\psi_{71}(t) \\ \psi_{71}(t)=\dfrac{2}{a}\left(\displaystyle\int_0^b+\int_d^e\right)r(\alpha)\left[\phi_1(\alpha,t)+\phi_0(\alpha,t)\right]d\alpha \qquad ,0\le t<b \\ h_4(t)=-\dfrac{2}{a}\psi_7(t) \qquad , b<t<e \end{array}\right]$$

$$\left.\begin{array}{l} g_6(t)=\dfrac{2}{a}\psi_8(t) \quad ,\psi_8(t)=r(t)\psi_{81}(t) \\ \psi_{81}(t)=\dfrac{2}{a}\left(\displaystyle\int_0^b+\int_d^e\right)r(\alpha)\left[\phi_1(\alpha,t)+3\phi_0(\alpha,t)\right]d\alpha \qquad ,0\le t<b \end{array}\right]$$

$$h_6(t)=-\frac{2}{a}\psi_8(t) \qquad , d<t<e$$

Thus

$$g(t)=\frac{2}{a}r(t)\psi_9(t) \quad ,0\le t<b$$

$$\psi_9(t) \;=\left[\Delta_{01}(t)+2e^{-2q\delta_1}\left(\psi_{61}(t)+e^{-2q\delta_1}\psi_{71}(t)\right)+e^{-4q\delta_1}\psi_{81}(t)\right]$$

$$h(t)=-\frac{2}{a}r(t)\psi_9(t) \qquad , b<t<e$$

Thus substituting (5.9) in (4.1) and (5.10) in (4.2) and then evaluation of integrals by numerical method will give temperature.

The substitution of (5.9) - (5.10) into (4.9) and thus numerical evaluation of integrals will give us K_b , K_d ,K_e from (4.13).

DISCUSSION AND CONCLUSION

The problem of disturbance in heat distribution caused due to a Griffith crack in an isotropic rectangle with most general boundary condition had been discussed. The general problem is reduced to Fredholm integral equation of second kind. The solution of Fredholm integral equation is obtained by approximation the kernel for prescribed (known) temperature and flux at boundary. Method can be used for most general conditions also. We used Fourier series method with the principle of cross-linear superposition. It is observed that the temperature is smooth at crack tips while flux is singular. It has Cauchy type of singularity.

REFERENCES

1. K. S. Parihar, (1970), "Some Triple Trigonometrical Series Equation [163] and Their Applications" Proc.Roy. Soc. Edinb. A-69, pp. 245-265.
2. Umesh Chandra, (1989), "Two dimensional mixed boundary value problems in a strip", A Ph.D. Thesis, Agra University.
3. P.S.Sorout,(1992),"Mixed boundary value problems in orthotropic medium due to heated wedge", A Ph.D. Thesis, Agra University.
4. Anil Kumar Singh,(1996),"The crack opening due to heated wedge in an Isotropic rectangle", A Ph.D. Thesis, Agra University.
5. Harendra Singh,(2000),"The stress-intensity factor at crack tips opened by thermal stress in orthotropic in finite medium", A Ph.D. Thesis, Agra University.
6. Anjana Singh,(2001),"The stress-intensity factor due to n-Griffith-cracks opened by thermal stress in orthotropic medium", A Ph.D. Thesis, Agra University.
7. Sunil Kr. Yadav,(2000),"Distribution in temperature distribution due to presence of cracks in encased cylinders", A Ph.D. Thesis, Agra University.
8. K.S. Parihar,(1970) "Some Triple Trigonometrical Series Equation and Their Applications"Proc. Roy. Soc. Edinb. A69 pp 245-265
9. Hany H.Sherif; Nasser M.El-Maghraby,(2005),"A mode-I crack problem for an infinite space in generalized thermoelasticity."Int.J.of Thermal stress.vol 28, issue 5, pp 465-484.
10. L.M.Brock,(2004),"three cases of rapid crack extension in a transversely isotropic coupled thermoelastic field."Int.J.of Thermal stress.vol 27, issue 1 ,pp 75-94.
11. Davresh Hasanyan; Liviu Librescue; Zhanming Qin;Richard Young,(2005)," Thermoelsatic cracked plates carrying nonstationary elastic current." Int.J.of Thermal stress.vol 28, issue 6& 7 ,pp 729-745.
12. Bansal K.K.,Sharma M.K.[2015] A Comparative Study of Reliability Analysis of a Non-Series Parallel Network. *International Journal of Education and Science Research Review Vol.2(6)*
13. Sharma M.K. Bansal K.K. [2015] Fuzzy Analysis Of Shortest Job First. *International Journal of Engineering Research & Management Technology Vol.2(3)*
14. Sharma M.K. Bansal K.K. [2015] A Comparative Study of Reliability Analysis of a Non-Series Parallel Network. *International Journal of Education and Science Research Review Vol.2(6)*
15. F.Suat Kadioglu,(2006)," circumferentially cracked biomaterial hollow cylinder under mechanical and transient thermal loading." Int.J.of Thermal stress.vol 29, issue 12,pp 1073- 1106.
16. F.Suat Kadioglu,(2008)," circumferentially cracked hollow cylinder under thermal shock."Int.J.of Thermal stress.vol 31, issue 11,pp 1056-1078.
17. B.P.Fillery; X.Z.Hu,(2010)," compliance based assessment of stress intensity factor in cracked plates with finite boundary restraint: application to thermal shock part-I." Int.J.of Thermal stress.vol 33, issue 7,pp 619-645.
18. Brock, L.M., (2004), "three cases of rapid crack extension in a transversely isotropic coupled thermo elastic field." Int. J. of Thermal stress.vol 27, issue1, pp 75-94.